Solutions to 2nd Order Differential Equations

A second-order differential equation is defined by the form:

$$ay'' + by' + cy = 0$$

where a, b, c are all constants. To find the solution to a 2nd order DE, you need to first determine the characteristic equation. The characteristic equation is given by the form:

$$ar^2 + br + c$$

Plugging in your variables will allow you to solve the quadratic equation for r. In the general case, if both roots are real numbers and different from each other, the solution for the differential equation will then be:

$$y(t) = C_1e^{r_1t} + C_2e^{r_2t}$$

However, what if there are complex or repeated roots? For complex roots, the solution is a bit different. Solving a quadratic with complex roots will give you a solution in the form

$$r = \alpha \pm \beta i$$

This is derived from the quadratic formula. Plugging this r into y(t) will get you a solution involving complex numbers. By using Euler's formula, however, the equation can be rewritten into a real form. This ends up becoming the solution:

$$y(t) = C_1e^{\alpha t}\cos(\beta t) + C_2e^{\alpha t}\sin(\beta t)$$

Finally, there's the case with repeated roots. If the solution to your characteristic equation gives repeated roots, then the solution to the differential equation is:

$$y(t) = C_1e^{rt} + C_2te^{rt}$$